Math 42-Number Theory
Problem Set #10: Review
Due Tuesday, May 3, 2011

1. Describe all solutions $x, y \in \mathbb{Z}$ satisfying $95x + 28y = 1$.

2. Find a solution (X, Y) with X and Y in $\mathbb{Z}[i]$ to the equation $(11 + 5i)X + (4 + 7i)Y = 1$.

3. Use quadratic reciprocity to decide the number of solutions to each of the following equations (without actually solving). Note that $5029 = 47 \cdot 107$.

(a) $x^2 \equiv 5 \mod 5029$
(b) $x^2 \equiv 14 \mod 5029$
(c) $x^2 \equiv 6 \mod 5029$

4. For all of the equations from problem 3 that have solutions, find those solutions by:

- finding a generator mod 47 and mod 107 (do this by hand and show all work),
- making logarithm tables and solving equations mod 47 and mod 107,
- using the Chinese Remainder Theorem.

5. Find a solution to $x^2 - 13y^2 = 1$. Use this solution to find two more solutions.

6. Give an example of a system without unique factorization into primes. Explain your example by giving two different factorizations of the same number.

7. When is 17 a square mod p?

8. Prove that $a^{(p-1)/2} \equiv \left(\frac{a}{p}\right) \mod p$ for $a \in U_p$ and p an odd prime.

9. Prove that $a^{p-1} \equiv 1 \mod p$ for $a \in U_p$ and p a prime.

10. Prove using induction that $1 + 3 + 5 + 7 + \ldots + (2n - 1) = n^2$ for any natural number n.

11. Prove that there are infinitely many primes. (Hint: suppose there are finitely many, and try to construct a new prime, thus obtaining a contradiction.)